# $\beta_{\rm eff}$ calculations using JEFF-3.1 nuclear data

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Petten, October 7, 2005

21616/05.69454/P FAI/SVDM/MB

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16 page(s)				approved :	R.P.C. Schram	
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#### Abstract

Calculations were performed with MCNP-4C3 to validate the delayed neutron data of the new JEFF-3.1 nuclear data library.

### Keywords

MCNP-4C3 JEFF-3.1 nuclear data effective delayed neutron fraction

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## **1. Introduction**

Recently the new nuclear data library JEFF-3.1 has been made available by the NEA. The delayed neutron data in this library is new. For the first time, 8 time groups have been used to represent these data, instead of the customary 6 time groups [1].

To validate the results that can be obtained with the new delayed neutron data, we have calculated  $\beta_{\text{eff}}$  for many systems for which measurements have been reported in the open literature. The results of these calculations are reported here.

The results in this report have been obtained using MCNP-4C3 [2], extended with a method to calculate  $\beta_{eff}$  [3]. The standard versions of MCNP-4C3, MCNP-5, and MCNPX can all work with delayed neutron data in 8 time groups. However, because these codes do not calculate a value for  $\beta_{eff}$ , we included our own  $\beta_{eff}$  method into MCNP-4C3. This method has been used earlier to calculate  $\beta_{eff}$  for other nuclear data libraries, viz. JEFF-3.0, ENDF/B-VI.8, and JENDL-3.3 [4].

## 2. Benchmark systems

We have searched in the literature for measurements of the effective delayed neutron fraction, the result of which is listed below. We will use these experiments as benchmarks for the calculation of  $\beta_{\text{eff}}$  on the basis of JEFF-3.1 nuclear data. For some systems we have found experimental values for the parameter  $\alpha$ , which is linked to  $\beta_{\text{eff}}$  through  $\alpha = [k(1 - \beta_{\text{eff}}) - 1]/l$ , where *l* is the prompt neutron life time. All systems described below are at delayed criticality, so that the parameter we can compare with is the value  $\alpha_{\text{dc}} = \alpha(k = 1) = -\beta_{\text{eff}}/l$ .

When not stated explicitly, the MCNP [2] model for the experiment was taken, without modifications, from the ICSBEP data [5]. Where possible, the ICSBEP identification is given in brackets after the benchmark name.

• Godiva (heu-met-fast-001)

A bare sphere of highly enriched (94 wt%) uranium.

- Jezebel (pu-met-fast-001)
- A bare sphere of plutonium (95 at% Pu-239).Skidoo (u233-met-fast-001)

A bare sphere of uranium, of which 98 at% U-233.

- **Topsy** (Flattop 25, heu-met-fast-028) A highly enriched (93 wt%) uranium sphere surrounded by a thick reflector of normal uranium. Experimental results are given in Ref. [6].
- **Popsy** (Flattop-Pu, pu-met-fast-006) A plutonium (94 wt% Pu-239) sphere surrounded by a thick reflector of normal uranium. Experimental results are given in Ref. [6].
- Flattop 23 (u233-met-fast-006) A uranium (98 at% U-233) sphere surrounded by a thick reflector of normal uranium. Experimental results are given in Ref. [6].
- **Big Ten** (ieu-met-fast-007) A large, mixed-uranium-metal cylindrical core with 10% average U-235 enrichment, surrounded by a thick reflector of depleted uranium [7].
- ZPR (heu-met-inter-001, ieu-met-fast-010, mix-met-fast-011 case 1, pu-met-inter-002)
   Four cores in the Zero Power Reactor at ANL. The first one is a highly enriched uranium/iron benchmark, reflected by steel. The second is a heterogeneous cylindrical core of uranium (average enrichment 9%). The third has plutonium/uranium/zirconium fuel, reflected by graphite. The last core had heterogeneous plutonium metal fuel with carbon/stainless steel dilutions, and a steel reflector. Measured values for β<sub>eff</sub> are given in e.g. Ref. [8].
- SNEAK (cores 7A, 7B, 9C1, and 9C2)
   Measurements of β<sub>eff</sub> in four unmoderated PuO<sub>2</sub>-UO<sub>2</sub> cores, surrounded by a depleted uranium reflector [9]. One core, 9C1, had only uranium as fuel. The 9C2 core was diluted with sodium. MCNP models were built based on the R-Z model descriptions in Ref. [9].

• Masurca (cores R2 and ZONA2)

Measurements of  $\beta_{eff}$  by several international groups in two unmoderated cores, viz. R2 and ZONA2 [10]. Core R2 had of ~ 30% enriched uranium as fuel, whereas ZONA2 had both plutonium and depleted uranium. Both cores were surrounded by a 50-50 UO<sub>2</sub>-Na mixture blanket, and by steel shielding. MCNP models were built based on the R-Z model descriptions

in Ref. [10].

• FCA (cores XIX-1, XIX-2, and XIX-3)

Measurements of  $\beta_{eff}$  by several international groups in three unmoderated cores in the Fast Critical Assembly [10]. One core had highly enriched uranium, one had plutonium and natural uranium, and the third one had plutonium as fuel. The cores were surrounded by two blanket regions, one with depleted uranium oxide and sodium, and another one with only depleted uranium metal. MCNP models were built based on the R-Z model descriptions in Ref. [10].

• TCA (related to leu-comp-therm-006)

A light water moderated low-enriched  $UO_2$  core in the Tank-type Critical Assembly. From the description of this experiment in Ref. [11] it is clear that this experiment is closely related to benchmark leu-comp-therm-006 [5]. We have taken the MCNP input decks given in Ref. [5], and changed the loading pattern, the water height and lattice pitch.

- IPEN/MB-01 (related to leu-comp-therm-077)
   Measurement of β<sub>eff</sub> in the research reactor IPEN/MB-01, with a core consisting of 28 × 26 UO<sub>2</sub> (4.3% enriched) fuel rods inside a light water filled tank [12]. An MCNP input deck was made available by the authors of Ref. [12].
- Winco slab tanks (related to heu-sol-therm-038 case 5)

Measurement of  $\alpha$  in the Westinghouse Idaho Nuclear Company Slab Tank Assembly. The experiment consisted of two thin coaxial slab tanks with 93% enriched uranyl nitrate solution. From the description of this experiment in Ref.[13] it is clear that this experiment is closely related to heu-sol-therm-038, case 5 [5]. We have taken the MCNP input deck given in Ref. [5], and removed the stainless steel absorber between the two slab tanks.

• Stacy (leu-sol-therm-004, -007, -016, -021)

Measurements of  $\beta_{\text{eff}}/l$  in uranyl nitrate solution (10 % enrichment) in several cores in the STACY facility. From the description of these experiments in Ref. [14], one can identify several experiments that have been included in the criticality benchmark collection [5].

• Sheba (core II)

Measurement of  $\beta_{\text{eff}}/l$  in a critical assembly vessel, filled with 5% enriched uranyl fluoride, UO<sub>2</sub>F<sub>2</sub>, the Solution High-Energy Burst Assembly [15]. The vessel had a cylindrical shape, and there was no reflector. An MCNP model was built from scratch.

• SHE-8

Measurement of  $\beta_{\text{eff}}/l$  in a split table type critical assembly called Semi-Homogeneous Assembly [16]. The core was shaped in a hexagonal prism, with graphite matrix tubes and graphite rods. There was no axial reflector. The central region in core 8 consisted of 73 fuel rods with 2.9% enriched UO<sub>2</sub> dispersed in graphite. An MCNP model was built from scratch.

• **Proteus** (core 5)

Measurement of  $\beta_{\text{eff}}/l$  in a graphite reflected pebble bed reactor, containing uranium-carbon fuel pebbles (16.7% enrichment) and graphite moderator pebbles. As the reactor was operated below 1 kW, no coolant was needed. An MCNP model was made available by the authors of Ref. [17].

For Godiva, Jezebel, and Skidoo, both Keepin [18] and Paxton [6] give experimental values for  $\beta_{eff}$ . Although these values are not identical, the differences are small and have no significant impact on the conclusions drawn in this paper. Therefore we will use the numbers given by Keepin, because this is the commonly used reference.

For Big Ten, an experimental value  $\alpha = -(1.17\pm0.01)\times10^5$  s<sup>-1</sup> is given by Paxton [6]. Based on a

calculation of  $\beta_{\text{eff}} = 720 \text{ pcm}$ , he estimates the prompt neutron life time to be  $6.15 \times 10^{-8}$  s, which is consistent with our calculation of  $6.15 \times 10^{-8}$  s within a fraction of a percent. Therefore we feel it is justified in this case to compare with the value of  $\beta_{\text{eff}} = 720 \text{ pcm}$  as if it were determined by experiment.

For the Stacy, Winco, SHE-8, Sheba-II, and Proteus experiments, we will compare with the  $\alpha$  values given in the respective references, because that is the measured quantity. Also, for the Winco experiment, there is some uncertainty in deriving a value for  $\beta_{eff}$  from the measured  $\alpha$ . The comparison in the next section is done by dividing the calculated  $\beta_{eff}$  by the prompt neutron fission life time. This life time is calculated by MCNP by default, and is given in the output as the 'fission lifespan' (see the discussion of life time estimation in section 2.VIII.B of the manual [2]).

## 3. Results

The results for the Sheba-II, SHE-8, TCA and Winco experiments should be viewed with some caution, since the preparation of the MCNP models for these experiments involved interpretation on our part, based on the references given earlier. However, since the computational results for these cases are close to the experimental values, we judge the models to be appropriate for calculating  $\beta_{\text{eff}}$  and  $\alpha$ .

	Experiment	Calcu	ulation	C/E (JEFF-3.1)
		JEFF-3.0	JEFF-3.1	
TCA	771±17	817±9	793.4±1.4	$1.029 \pm 0.002$
IPEN	$742 \pm 7$	$788 \pm 4$	771.7±4.1	$1.040 {\pm} 0.005$
Masurca_R2	721±11	735±7	$728.9{\pm}6.9$	$1.011 \pm 0.009$
Masurca_Z2	349±6	$358\pm5$	$356.2{\pm}4.6$	$1.021 \pm 0.013$
FCA-XIX-1	$742 \pm 24$	$767\pm8$	749.7±7.5	$1.010 {\pm} 0.010$
FCA-XIX-2	364±9	387±5	383.7±4.9	$1.054{\pm}0.013$
FCA-XIX-3	$251 \pm 4$	$255 \pm 4$	$250.3 {\pm} 4.1$	$0.997 {\pm} 0.016$
Sneak-9C1	$758 \pm 24$	757±7	743.8±7.0	$0.981 {\pm} 0.009$
Sneak-7A	395±12	381±5	377.3±4.8	$0.955 {\pm} 0.013$
Sneak-7B	429±13	$447 \pm 5$	$434.2 {\pm} 5.0$	$1.012 \pm 0.012$
Sneak-9C2	426±19	395±5	388.0±4.9	$0.911 {\pm} 0.013$
Zpr-Heu	667±15	696±9	$685.8{\pm}8.8$	$1.028 {\pm} 0.013$
Zpr-U9	$725 \pm 17$	769±9	$747.7 \pm 8.3$	$1.031 {\pm} 0.011$
Zpr-Mox	381±9	373±6	$369.8 {\pm} 5.1$	$0.971 {\pm} 0.014$
Zpr-Pu	$222 \pm 5$	243±5	$241.3 \pm 5.2$	$1.087 \pm 0.022$
BigTen	720±7	$765\pm7$	735.3±6.4	$1.021 \pm 0.009$
Godiva	659±10	677±8	$669.4{\pm}4.0$	$1.016 {\pm} 0.006$
Topsy	665±13	666±8	659.5±3.9	$0.992 {\pm} 0.006$
Jezebel	$194{\pm}10$	$200\pm5$	$201.8 {\pm} 2.2$	$1.040 {\pm} 0.011$
Popsy	276±7	$283\pm5$	$282.5 {\pm} 2.5$	$1.024 {\pm} 0.009$
Skidoo	290±10	$284 \pm 5$	309.1±2.7	$1.066 {\pm} 0.009$
Flattop	360±9	343±6	361.9±2.8	$1.005 {\pm} 0.008$

Table 3.1 The experimental and calculated  $\beta_{eff}$  (in pcm). The uncertainty for C/E includes only the statistical uncertainty of the calculation. This corresponds to the error bar in Fig. 3.1. The experimental uncertainty range is shown with dashed lines in Fig. 3.1.

Concerning the Winco slab tank experiment, another remark is in order. Our calculation of  $\beta_{eff}$  yields a value of 845 ± 4 pcm, contradicting the value 1500 ± 120 calculated in Ref. [13], based on the experimental  $\alpha$ -value and other experimental information, not involving the prompt neutron fission life time. However, in Ref. [13] it is noted that its calculated value is higher than the expected value of roughly 900 pcm, the reason for which was not well understood. Our value for  $\alpha$  is close to the experimental value.

	Experiment	Calc	Calculation	
		<b>JEFF-3.0</b>	JEFF-3.1	
Proteus	$3.60 {\pm} 0.02$	$3.82 {\pm} 0.05$	$3.70 {\pm} 0.02$	$1.028 {\pm} 0.005$
SHE-8	$6.53 {\pm} 0.34$	$6.36 {\pm} 0.08$	$6.21 \pm 0.15$	$0.951 {\pm} 0.024$
Sheba-II	$200.3 \pm 3.6$		$203.79 \pm 1.38$	$1.017 {\pm} 0.007$
Stacy-029	$122.7 \pm 4.1$	$122.2{\pm}2.6$	$120.76 {\pm} 0.75$	$0.984{\pm}0.006$
Stacy-033	$116.7 \pm 3.9$	$129.5 \pm 2.4$	$114.11 \pm 0.72$	$0.978 {\pm} 0.006$
Stacy-046	$106.2 \pm 3.7$	$109.3 \pm 2.2$	$104.77 {\pm} 0.66$	$0.987 {\pm} 0.006$
Stacy-030	$126.8 {\pm} 2.9$	$129.8 {\pm} 2.7$	$129.17 {\pm} 0.81$	$1.019 {\pm} 0.006$
Stacy-125	$152.8 {\pm} 2.6$	$162.3 \pm 2.4$	$156.38 {\pm} 1.00$	$1.023 {\pm} 0.006$
Stacy-215	$109.2 \pm 1.8$	114.6±2.3	$109.28 {\pm} 0.68$	$1.001 {\pm} 0.006$
Winco	$1109.3 \pm 0.3$	1169.±13.	$1134.98 {\pm} 5.54$	$1.023 {\pm} 0.005$

Table 3.2 The experimental and calculated  $\alpha$  (in  $s^{-1}$ ). The uncertainty for C/E includes only the statistical uncertainty of the calculation. This corresponds to the error bar in Fig. 3.1. The experimental uncertainty range is shown with dashed lines in Fig. 3.1.

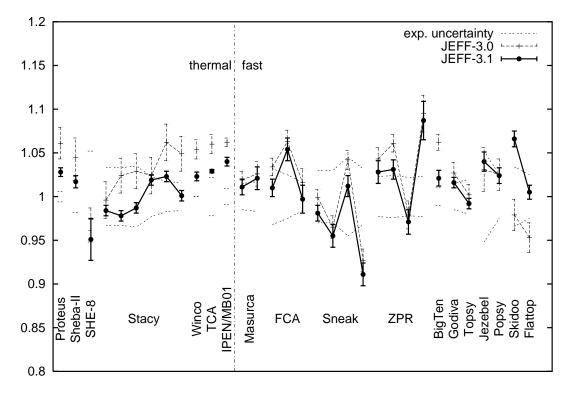


Figure 3.1 C/E for  $\beta_{eff}$  (or  $\beta_{eff}/l$ , see text) for many benchmark systems. The systems are roughly ordered with respect to the average energy at which fission takes place, from low energy (left) to high (right).

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## APPENDIX A. JEFF-3.1 results for $\beta_{eff}$ and $\beta_0$ in 8 groups

In the JEFF-3.1 nuclear data library the delayed neutron data have been represented using 8 time groups. Since this is the first time this representation has been used, it is deemed useful to list the results of our calculations per time group, for reference.

	Godiva		Jez	Jezebel		doo
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$
1	$24.2 \pm 0.8$	$20.9 {\pm} 0.5$	$6.5 \pm 0.4$	$6.4 \pm 0.3$	$26.6{\pm}0.8$	$22.4{\pm}0.5$
2	$95.4{\pm}1.5$	93.0±1.0	$45.4{\pm}1.0$	$47.2 {\pm} 0.7$	46.1±1.0	$44.1 {\pm} 0.7$
3	$69.4 \pm 1.3$	$61.5 {\pm} 0.8$	$18.6 {\pm} 0.7$	$19.2 {\pm} 0.4$	$44.4{\pm}1.0$	$38.0{\pm}0.6$
4	132.6±1.8	124.6±1.1	$30.9 \pm 0.8$	$31.3 {\pm} 0.6$	62.3±1.2	$56.4{\pm}0.8$
5	$208.9 \pm 2.2$	$198.5 {\pm} 1.4$	68.9±1.3	$71.0{\pm}0.8$	93.6±1.5	86.1±0.9
6	$62.7 \pm 1.2$	$59.5 \pm 0.8$	$8.4{\pm}0.4$	$8.5 {\pm} 0.3$	$12.0 {\pm} 0.5$	$10.9 \pm 0.3$
7	59.3±1.2	$55.5 \pm 0.7$	$19.2 {\pm} 0.7$	$20.1 {\pm} 0.5$	19.4±0.7	$17.0 {\pm} 0.4$
8	$17.0 \pm 0.7$	$16.0 {\pm} 0.4$	$3.8 {\pm} 0.3$	$3.7{\pm}0.2$	$4.8 {\pm} 0.4$	$3.9{\pm}0.2$

Table A.1	The partial $\beta_{eff}$ and $\beta_{0}$	for Godiva	, Jezebel and S	kidoo (in pcm).
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	Topsy		Po	opsy	Flattop-23	
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$
1	$21.1 \pm 0.7$	$20.8\pm0.5$	$6.1 \pm 0.4$	9.0±0.3	21.3±0.7	$20.6 \pm 0.5$
2	$85.5 \pm 1.4$	$107.6 {\pm} 1.0$	$47.3 \pm 1.0$	$76.9{\pm}0.9$	$47.5 {\pm} 1.0$	$73.2 \pm 0.9$
3	$60.1 \pm 1.2$	$61.2 {\pm} 0.8$	$19.9 {\pm} 0.7$	$31.0 {\pm} 0.6$	39.4±1.0	$44.4 {\pm} 0.7$
4	$125.8 {\pm} 1.7$	$147.7 \pm 1.2$	$40.6 {\pm} 0.9$	$79.9{\pm}0.9$	66.4±1.2	99.3±1.0
5	$202.7 \pm 2.2$	249.9±1.6	90.6±1.4	170.7±1.3	$107.8 {\pm} 1.5$	$175.1 \pm 1.3$
6	75.3±1.3	$107.9 \pm 1.0$	$30.5 \pm 0.8$	81.5±0.9	$33.9 {\pm} 0.8$	$79.7 {\pm} 0.9$
7	64.2±1.2	84.3±0.9	$33.1 {\pm} 0.8$	$65.8 \pm 0.8$	$31.0 {\pm} 0.8$	$60.5 {\pm} 0.8$
8	$24.8 {\pm} 0.7$	$40.4{\pm}0.6$	$14.3 {\pm} 0.5$	$37.7 {\pm} 0.6$	$14.7 {\pm} 0.6$	$36.9 {\pm} 0.6$

Table A.2 The partial  $\beta_{eff}$  and  $\beta_0$  for Topsy, Popsy, and Flattop-23 (in pcm).

group g	$\beta_{{ m eff},g}$	$eta_{0,g}$
1	$15.4 {\pm} 0.9$	$20.1 \pm 0.7$
2	$97.8 \pm 2.3$	$120.5 \pm 1.8$
3	$52.0 \pm 1.7$	63.2±1.3
4	125.7±2.6	155.7±2.1
5	$221.7 \pm 3.5$	279.1±2.8
6	$101.4{\pm}2.3$	130.7±1.9
7	$77.8 \pm 2.1$	98.6±1.7
8	43.3±1.6	54.6±1.2

Table A.3 The partial  $\beta_{eff}$  and  $\beta_0$  for Big Ten (in pcm).

	Pu		М	Mox		HEU		U9	
g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	
1	$8.2{\pm}1.0$	$7.7 {\pm} 0.6$	$7.9{\pm}0.7$	$9.0 {\pm} 0.5$	$25.9 \pm 1.7$	$21.7 \pm 1.0$	$15.7 \pm 1.2$	19.5±0.9	
2	$52.5 \pm 2.4$	$50.2 \pm 1.5$	$63.2 \pm 2.1$	62.4±1.3	$98.2 \pm 3.4$	$98.7 \pm 2.1$	98.1±3.0	$118.0{\pm}2.3$	
3	$22.3{\pm}1.6$	$21.8 \pm 1.0$	30.6±1.4	$29.6 {\pm} 0.9$	$69.9 {\pm} 2.8$	$65.0 \pm 1.7$	53.6±2.3	$65.8 {\pm} 1.7$	
4	$34.9{\pm}2.0$	33.4±1.2	$59.3 \pm 2.0$	58.5±1.3	136.6±3.9	$133.3 \pm 2.4$	$124.7 \pm 3.3$	$158.7 \pm 2.7$	
5	86.3±3.1	83.1±1.9	$118.3 \pm 2.8$	119.2±1.8	213.9±4.9	$207.2 \pm 3.0$	226.4±4.5	$277.7 \pm 3.5$	
6	$7.9{\pm}0.9$	$8.9{\pm}0.6$	$37.9 \pm 1.7$	35.3±1.0	$65.6 {\pm} 2.8$	$62.0 \pm 1.7$	$105.3 \pm 3.1$	$128.8 {\pm} 2.4$	
7	$25.2{\pm}1.7$	$21.4{\pm}1.0$	39.4±1.7	36.7±1.0	$57.5 \pm 2.6$	56.6±1.6	$80.5 \pm 2.7$	$96.8 {\pm} 2.1$	
8	$3.9{\pm}0.7$	$3.9{\pm}0.4$	13.3±1.0	$13.9 {\pm} 0.6$	$18.4{\pm}1.5$	16.1±0.8	43.5±2.0	$52.3 \pm 1.5$	

Table A.4 The partial  $\beta_{eff}$  and  $\beta_0$  for ZPR-Pu, ZPR-Mox, ZPR-HEU, and ZPR-U9 (in pcm).

	TC	CA	IPEN/MB-01		
group g	$\beta_{{ m eff},g}$	$\beta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	
1	24.3±0.2	$21.4{\pm}0.1$	$26.3 {\pm} 0.8$	$22.9 \pm 0.5$	
2	$115.1 {\pm} 0.5$	$106.0 {\pm} 0.3$	$111.3 \pm 1.6$	$103.7 \pm 1.0$	
3	$68.7 {\pm} 0.4$	$61.0 {\pm} 0.2$	$70.1 \pm 1.2$	$61.5 {\pm} 0.8$	
4	$150.5 {\pm} 0.6$	136.9±0.4	$148.4{\pm}1.8$	$136.2 \pm 1.2$	
5	$256.0{\pm}0.8$	$234.3 {\pm} 0.5$	$250.0{\pm}2.3$	$229.0{\pm}1.5$	
6	83.6±0.4	75.6±0.3	76.5±1.3	$69.7 \pm 0.9$	
7	69.7±0.4	63.3±0.3	66.5±1.2	$60.8 \pm 0.8$	
8	$25.6 {\pm} 0.2$	$23.5 \pm 0.2$	$22.5 \pm 0.7$	21.1±0.5	

Table A.5 The partial  $\beta_{eff}$  and  $\beta_0$  for TCA and IPEN/MB-01 (in pcm).

	R	2	ZONA2		
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	
1	21.9±1.2	21.3±0.8	6.5±0.6	8.3±0.5	
2	$102.1 \pm 2.6$	$110.1 \pm 1.8$	57.9±1.8	$71.6 \pm 1.4$	
3	$63.5 \pm 2.1$	$64.8 \pm 1.3$	$21.8 \pm 1.1$	$27.9 \pm 0.9$	
4	132.6±2.9	$142.2{\pm}2.0$	$50.1 \pm 1.7$	$62.9 \pm 1.3$	
5	222.7±3.8	241.6±2.6	114.9±2.6	$146.6 {\pm} 2.0$	
6	$87.8 {\pm} 2.4$	$96.8 {\pm} 1.6$	$44.0 \pm 1.6$	$60.7 \pm 1.3$	
7	69.8±2.1	75.7±1.5	40.3±1.5	$55.8 \pm 1.2$	
8	$28.4{\pm}1.4$	$33.5 \pm 1.0$	$20.8 {\pm} 1.2$	$25.9 {\pm} 0.8$	

Table A.6 The partial  $\beta_{eff}$  and  $\beta_0$  for Masurca-R2 and Masurca-ZONA2 (in pcm).

	FCA-XIX-1		FCA-2	FCA-XIX-2		XIX-3
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$\beta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$
1	$24.4{\pm}1.4$	$20.9 \pm 0.8$	$7.2 \pm 0.7$	$8.2{\pm}0.5$	$6.0{\pm}0.6$	8.1±0.5
2	$106.4 \pm 2.9$	$101.9 \pm 1.7$	$60.2 \pm 1.9$	$69.8 {\pm} 1.4$	$50.0{\pm}1.8$	56.7±1.3
3	$75.0{\pm}2.3$	$68.2 \pm 1.4$	$26.5 \pm 1.3$	$30.9 {\pm} 0.9$	$22.3 \pm 1.3$	$24.4 {\pm} 0.8$
4	$147.5 \pm 3.3$	136.7±2.0	$59.5 \pm 2.0$	68.9±1.4	39.9±1.6	47.3±1.1
5	$234.2 \pm 4.2$	$227.4{\pm}2.5$	122.5±2.8	$146.8 {\pm} 2.0$	$82.8 {\pm} 2.4$	99.4±1.7
6	$73.5 \pm 2.4$	$77.5 \pm 1.5$	$45.7 \pm 1.7$	56.6±1.3	$16.7 \pm 1.0$	26.3±0.9
7	68.1±2.3	67.5±1.4	42.1±1.6	52.6±1.2	26.4±1.3	31.9±0.9
8	$20.6 \pm 1.2$	$23.4 {\pm} 0.8$	19.9±1.1	26.5±0.9	6.3±0.6	$10.4 {\pm} 0.5$

Table A.7 The partial  $\beta_{eff}$  and  $\beta_0$  for FCA-XIX-1, FCA-XIX-2, and FCA-XIX-3 (in pcm).

	7A		7B		9C2		9C1	
g	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	$\beta_{\mathrm{eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$
1	$6.5 {\pm} 0.6$	$8.8{\pm}0.5$	$7.6 {\pm} 0.6$	$10.9 {\pm} 0.5$	$6.9 \pm 0.7$	9.4±0.5	$20.8 \pm 1.2$	$22.3 \pm 0.8$
2	$59.6 \pm 1.9$	$75.4{\pm}1.4$	$64.6 \pm 1.9$	$80.8 {\pm} 1.5$	60.7±1.9	77.7±1.5	$101.7 {\pm} 2.6$	$110.5 \pm 1.8$
3	23.7±1.2	31.9±0.9	25.8±1.2	33.1±1.0	26.3±1.3	32.3±0.9	$62.8 {\pm} 2.1$	$62.3 \pm 1.3$
4	$56.7 \pm 1.9$	$76.9 \pm 1.5$	$63.6 {\pm} 2.0$	$82.7 \pm 1.5$	56.4±1.9	$77.1 \pm 1.5$	133.7±2.9	$147.6 {\pm} 2.0$
5	$120.4{\pm}2.7$	$166.1 \pm 2.2$	$136.4{\pm}2.8$	$178.0{\pm}2.2$	123.9±2.8	$165.7 \pm 2.1$	$225.6 \pm 3.8$	$251.6 \pm 2.6$
6	$47.0 \pm 1.7$	$71.8 \pm 1.4$	$60.5 \pm 1.9$	81.3±1.5	49.3±1.8	$71.0 \pm 1.4$	93.8±2.5	$105.9 \pm 1.7$
7	41.8±1.6	57.6±1.3	49.4±1.7	65.2±1.3	42.8±1.6	60.3±1.3	73.4±2.2	82.3±1.5
8	21.6±1.1	33.2±1.0	26.4±1.2	36.1±1.0	21.6±1.2	$32.2 \pm 0.9$	31.9±1.4	39.3±1.0

Table A.8 The partial  $\beta_{eff}$  and  $\beta_0$  for Sneak-7A, Sneak-7B, Sneak-9C2, and Sneak-9C1 (in pcm).

	029		033		046	
group g	$\beta_{{ m eff},g}$	$\beta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$
1	$25.4{\pm}0.9$	22.1±0.5	$24.5 \pm 0.8$	$21.7 \pm 0.5$	$24.5 \pm 0.8$	$22.5 \pm 0.5$
2	$113.5 \pm 1.8$	$102.8 {\pm} 1.0$	$111.3 \pm 1.8$	$104.0{\pm}1.0$	$107.1 \pm 1.7$	$102.3 \pm 1.0$
3	$68.7 \pm 1.4$	$61.3 {\pm} 0.8$	$64.6 \pm 1.4$	$60.0\pm0.8$	$66.1 \pm 1.4$	$61.0 {\pm} 0.8$
4	$143.6 {\pm} 2.0$	$130.8 {\pm} 1.2$	$143.6 {\pm} 2.0$	$131.0 \pm 1.2$	$141.7 {\pm} 2.0$	$131.5 \pm 1.2$
5	$241.2 \pm 2.7$	$220.0{\pm}1.5$	$243.7 \pm 2.7$	$221.5 \pm 1.5$	$237.0{\pm}2.6$	$219.8 {\pm} 1.5$
6	$70.2 \pm 1.4$	$61.4 {\pm} 0.8$	66.3±1.4	$61.4 {\pm} 0.8$	65.6±1.3	$60.9 {\pm} 0.8$
7	61.7±1.3	$54.9 {\pm} 0.7$	59.7±1.3	$54.8 {\pm} 0.7$	59.2±1.3	$53.9 {\pm} 0.7$
8	$17.4 \pm 0.7$	$16.4 {\pm} 0.4$	$17.7 \pm 0.7$	16.3±0.4	$17.3 \pm 0.7$	$15.8 {\pm} 0.4$

Table A.9 The partial  $\beta_{eff}$  and  $\beta_0$  for Stacy runs 029, 033, and 046 (in pcm).

	030		125		215	
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$
1	$25.0\pm0.9$	$22.5\pm0.5$	$25.9{\pm}0.9$	$22.2 \pm 0.5$	$24.4 {\pm} 0.9$	$21.9 \pm 0.5$
2	116.4±1.8	$103.3{\pm}1.0$	114.7±1.9	$101.8 {\pm} 1.0$	$109.4{\pm}1.8$	$101.7 \pm 1.0$
3	$69.9 \pm 1.4$	$61.5 {\pm} 0.8$	$71.6 \pm 1.5$	$62.5\pm0.8$	67.7±1.4	$62.1 \pm 0.8$
4	$145.8 {\pm} 2.1$	$130.5 \pm 1.2$	$149.7 \pm 2.1$	$131.7 \pm 1.2$	$141.7 {\pm} 2.0$	$130.9 \pm 1.2$
5	$248.6 {\pm} 2.7$	$221.2{\pm}1.5$	$251.3 \pm 2.8$	$220.2{\pm}1.5$	$240.5{\pm}2.6$	$218.5 {\pm} 1.5$
6	$67.2 \pm 1.4$	$61.4{\pm}0.8$	68.7±1.5	$61.8 {\pm} 0.8$	66.4±1.4	$60.4 {\pm} 0.8$
7	59.8±1.3	$54.5 \pm 0.7$	$64.5 \pm 1.4$	$55.7 \pm 0.8$	57.2±1.3	$53.4 {\pm} 0.7$
8	17.7±0.7	15.5±0.4	19.0±0.8	16.5±0.4	$18.2 \pm 0.7$	$16.2 \pm 0.4$

Table A.10 The partial  $\beta_{eff}$  and  $\beta_0$  for Stacy runs 030, 125, and 215 (in pcm).

	Wi	nco	Sheba-II		
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	
1	$29.9 \pm 0.8$	$21.8 \pm 0.3$	$24.9 {\pm} 0.9$	21.3±0.5	
2	$124.5 \pm 1.6$	$101.3 {\pm} 0.7$	$117.4{\pm}2.1$	$102.2 \pm 1.1$	
3	82.1±1.3	61.1±0.6	72.7±1.6	$62.5 {\pm} 0.8$	
4	$169.2 \pm 1.9$	$130.6 {\pm} 0.8$	$151.8 \pm 2.3$	$133.2 \pm 1.2$	
5	273.1±2.3	$215.8{\pm}1.0$	262.0±3.1	$227.8 {\pm} 1.6$	
6	78.0±1.3	$60.6 {\pm} 0.6$	72.5±1.6	$64.3 \pm 0.8$	
7	$68.7 \pm 1.2$	$54.2 \pm 0.5$	$62.4 \pm 1.5$	$56.7 {\pm} 0.8$	
8	19.7±0.6	15.8±0.3	19.2±0.8	$17.6 {\pm} 0.4$	

Table A.11 The partial  $\beta_{eff}$  and  $\beta_0$  for the Winco slab tanks and Sheba-II (in pcm).

	Pro	teus	SHE-8		
group g	$\beta_{{ m eff},g}$	$eta_{0,g}$	$\beta_{{ m eff},g}$	$eta_{0,g}$	
1	$23.2 \pm 0.7$	$22.0 \pm 0.5$	22.9±3.0	$20.8 \pm 1.5$	
2	$109.0 \pm 1.5$	$102.5 {\pm} 1.0$	$106.1 \pm 6.3$	$104.6 \pm 3.4$	
3	$67.1 \pm 1.2$	$61.4 {\pm} 0.8$	$74.0{\pm}5.8$	$62.1 \pm 2.6$	
4	$140.7 \pm 1.7$	$132.3 \pm 1.2$	$140.7 \pm 7.6$	$129.4 \pm 3.8$	
5	231.3±2.2	217.5±1.5	$249.1 \pm 10.1$	$220.1 \pm 5.0$	
6	$64.5 \pm 1.2$	$59.8 \pm 0.8$	$61.9 {\pm} 5.0$	$59.3 \pm 2.6$	
7	57.1±1.1	$54.4 {\pm} 0.7$	$55.6 \pm 4.6$	$55.3 \pm 2.5$	
8	$15.9 {\pm} 0.6$	$15.4 \pm 0.4$	$15.8 \pm 2.4$	$16.6 \pm 1.4$	

Table A.12 The partial  $\beta_{eff}$  and  $\beta_0$  for Proteus and SHE-8 (in pcm).