REMARKS ON KINETICS PARAMETERS OF A SUB-CRITICAL REACTOR FOR NUCLEAR WASTE INCINERATION

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Abstract

Accelerator driven systems (ADS) are being designed as nuclear waste incinerator in order to be complementary with the proposed geological disposal. When the ADS is based on a highly sub-critical reactor, some elementary concepts designed for critical reactors need to be reconsidered:

- How much sub-critical means highly sub-critical?
- Is the spallation source really external?
- Are the kinetics parameters depending on the neutron source?
- Can standard neutron noise techniques be used when the neutron source is not Poisson-like?

The article remarks the subtleties behind those questions aiming to clarify future experiments with spallation source focused to nuclear waste incineration.

1. Introduction

At present, there are 423 nuclear power plants (NPP) operating around the world. Most of them are light water reactors, being the PWR type the more abundant, 247 NPP, followed by the BWR type, 92 NPP. As a consequence, the 3 000 MW_{th} PWR type is used as a reference for roughly estimation of the actinide amounts the humankind has to deal with shortly.

The total nuclear power around the world amounts 351.7 GWe, and it seems to follow the Pareto's law for the Economy, in the sense that the 20% of the 31 countries where nuclear power is installed have the 75% of the power [1]. Therefore, the nuclear waste problem has different weight for each country and no common solutions are expected to be taken. Based on the global power, operating during 40 years – 330 days/year – the forecast waste for the year 2010 is approximately [2]:

Table 1. **The nuclear waste for the year 2010**

Total spent fuel	*281 600 t*
Plutonium isotopes	2 816 t
Neptunium	131 t
Minor actinides	113 t
Long lived fission fragments	
^{99}Tc	235 t
^{135}Cs	84 t
^{129}I	56 t

The case of Spain can be meaningful [3,4]. The nuclear power is 7.74 GWe, about the 2.1% of the world nuclear power. According to Table 1, the expected amount for plutonium is 60 tonnes roughly. This material may be regarded either as a waste to take care of, either a fissile material to extract energy from.

After fuel reprocessing, accelerator driven systems (ADS), based on a proton accelerator coupled to a highly sub-critical reactor, can be designed for the actinide and long lived fission fragment incineration, acting as a complementary solution for the nuclear waste problem [5]. If so, it has several advantages:

- The geological disposal will be cheaper.
- Help for public acceptance of a geological disposal.
- Nuclear waste is converted into fuel.

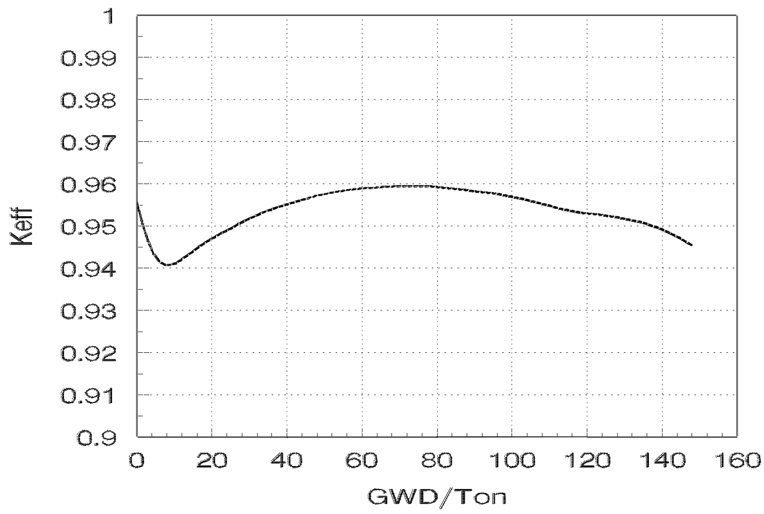
The last item comes from the energy content of the actinides, about 940 MWD/Kg. The economical income coming from the incineration of the actinides produced in Spain will cover the cost of the incinerator, hence, a priori is not a bad option for the nuclear waste problem. Nevertheless there are still many uncertainties around the ADS designs requiring detailed research. Highly sub-critical reactor driven by a spallation neutron source shows novel quests, in particular how to measure, control and even understand the k_{eff}, when the reactor is far from critical and what does it mean to be far from critical.

2. Sub-critical multiplication

The incineration rate should be high. This is achieved with a neutron flux higher than 5.10^{13} n $cm^{-2}s^{-1}$, corresponding to a typical PWR. According to the flux, the neutron population N can be calculated from $\phi = vN/V_R$, where v is the neutron mean velocity and V_R the reactor volume. In its turn, the global neutron population is: $N = S\ell/(1-k)$, where ℓ is the prompt neutron mean life and S the external neutron source intensity in n/s units [6]. Preliminary designs contemplate a magnitude about 5.10^{15} n cm^{-2} s^{-1} for ϕ; a flux high enough for fast incineration and breeding purposes – ^{233}U from ^{232}Th.

Such a flux needs a sub-critical reactor with a spallation neutron source. The closer to critical the higher the flux, but the restriction of non increasing the actual actinide inventory limits the fissile fuel chosen for the incinerator; as a consequence, preliminary design deals with k_{eff} about 0.95, which yields a multiplication of 20. Hence, the magnitude of S will be around 4.10^{17} n/s. Such a high intensity external neutron source is one of the key points of the incinerator. Preliminary designs are focused on the spallation of neutrons caused by 1.0 GeV proton beam coming from an accelerator; protons collide with a metallic target of high mass number -normally lead- causing neutrons to be expelled out from the nuclei until protons are stopped. The neutron source features are defined by the target and the accelerator current; some figures can define the order of magnitude: the operating accelerator current between 29 mA, the energetic neutron cost -in molten lead- about 36 MeV, the neutron spectrum is evaporation-type, similar to fission spectrum but with a higher energy tail.

Figure 1. **Evolution of k_{eff}**



Along the fuel burn-up the fuel composition will not remain constant, k_{eff} and the other kinetics parameters will drift slowly. That is a difference with critical reactors where the neutron flux increases in order to compensate the decreasing fuel density, keeping the power constant; the capability of keeping $k_{eff} = 1$ defines the burn-up period. In a sub-critical reactor the burn-up period is much longer because it is not limited by k_{eff}, but a drift of 1% in k_{eff} yields a drift of 20% in the flux, so in order to operate with constant power, it is important to measure k_{eff} accurately, and it is not easy for such a low multiplication with a non stationary source.

The relationship between this source and the k is even deeper. Being low the multiplication, the neutron shape function is affected by the source position; so, a change in the position of the spallation target means a change in k and the rest of kinetics parameters. And not only in the space domain, but in the time domain, because the spallation source can have a fast pulsed nature due to the accelerator, which in its turn affects to the flux shape normalisation, causing that static and dynamic reactivity measurements might differ. The above ideas are to be explored in the next sections.

3. External source and sub-criticity

In the case of a highly sub-critical reactor the k depends on the external neutron source. When close to critical state, this dependence is too weak and can be neglected. To argue that, the qualitative definitions of k is:

$$k = \frac{fission\ rate}{(absortion + leakage)\ rate}$$

according to the diffusion equation the leakage rate can be written as:

$$-D\Delta\phi = \nu\Sigma_f\phi - \Sigma_a\phi + S_V$$

where S_V stands for the stationary external source per volume unit and the other symbols have the habitual meaning. In terms of the buckling : $B^2 = (\nu\Sigma_f - \Sigma_a)/D = (k_\infty - 1)/L^2$:

$$k = \frac{\nu\Sigma_f\phi}{(\Sigma_a + DB^2)\phi + S_V}$$

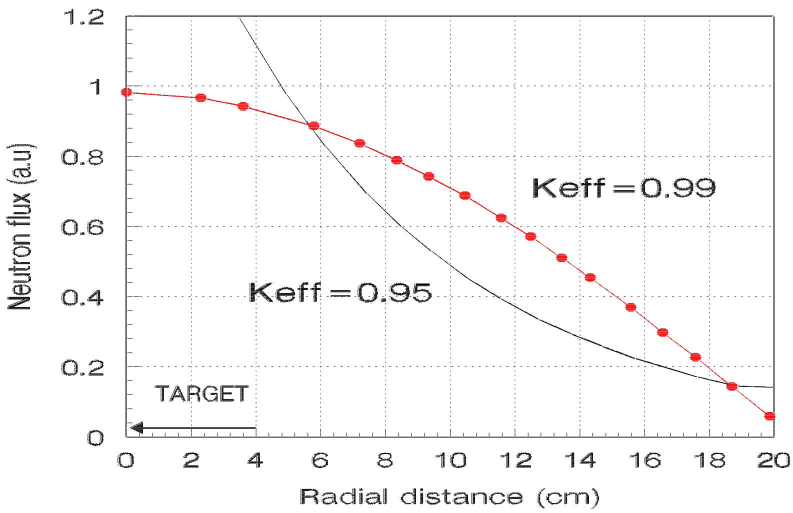
In critical reactors the term $DB^2\phi$ stands for the neutron leakage rate and DB^2 is regarded as a correction for Σ_a, but in sub-critical reactors the leakage rate is a function of the source which can be ignored when the source intensity is negligible compared with the fission rate, i.e. for large multiplication. Considering that ϕ is proportional to S_V it is clear that k does not depend on the power level.

In the Figure 2 two radial neutron flux distribution are plotted. They correspond to a cylindrical sub-critical reactor energy amplifier type with spallation source placed at the centre [5]. It is well known that the radial flux distribution is:

- Cosinus like $k_\infty > 1$.
- Straight line $k_\infty = 1$.
- Exponential like $k_\infty < 1$.

When the radial flux distribution is a straight line the reactor can be regarded as close to critical. In the example above it corresponds to $k_{eff} = 0.975$, nevertheless, many ADS are designed to operate at $k_{eff} = 0.95$, hence they will be highly sub-critical. As can be observed in Figure 2, the flux distribution is very much affected by spallation target position for the exponential like case; a change in the flux shape will be followed by a change in the sub-critical multiplication.

Figure 2. **Radial flux distribution for a energy amplifier type**



Let be the flux shape ψ be defined by: $\phi(r,t) = N(t)\psi(r,t)$ with the normalisation condition:

$$<W; \psi/v> = 1$$

where the bracket denotes the scalar product and W is the adjoint flux. Even when the k_{eff} is calculated by the usual procedure:

$$k_{eff} = \frac{<W; \nu\Sigma_f \psi>}{<W; (\Sigma_a - D\Delta)\psi>}$$

due to the laplacian operator, a change of ψ affects to k_{eff}. As a conclusion, for ADS the spallation source is not external and has to be considered for the calculation of the sub-critical multiplication factor.

The same argument applies for the rest of the kinetics parameter β and Λ. It must be distinguished between β as a quantity and β_{eff}. as a parameter. The quantity is a property of a given fissionable nucleus, so it does not depend on the spallation source; but the parameter does not correspond to any nucleus and it is calculated using the shape function [6,7], so it depends on the spallation source.

4. The stationarity of the spallation source

It is to be remarked that static and dynamic reactivity measurements might not coincide in a ADS with a spallation source. In the case of fast pulsing accelerator, the neutron flux shape function may be time dependent. If so, the Gyftopoulos term [8] should be added to the point kinetics equation causing that both type of measurement yield different results. Explicitly:

Following the normal procedure for point kinetics derivation [7] the shape function is defined with the condition:

$$<W;\frac{\psi}{v}>=1$$

so, for the global neutron population N(t):

$$\frac{\partial}{\partial t}[N<W;\frac{\psi}{v}>]=\frac{\partial N}{\partial t}$$

but when the transients are so fast that $<W;\psi/v>\neq 1$, because the shape function is time dependent, then:

$$\frac{\partial}{\partial t}[N<W;\frac{\psi}{v}>]=<W;\frac{\psi}{v}>\frac{\partial N}{\partial t}+N<W;\frac{1}{v}\frac{\partial \Psi}{\partial t}>$$

therefore a new term is carried into the point kinetics equations, now appearing as [8]:

$$\frac{dN}{dt}=(\frac{\rho-\beta}{\Lambda}-\lambda_S)N+\lambda C+S$$

$$\frac{dC}{dt}=\frac{\beta}{\Lambda}N-(\lambda+\lambda_S)C$$

where all the symbols have its habitual meaning except:

$$\lambda_S\equiv\frac{<W;\frac{1}{v}\frac{\partial\psi}{\partial t}>}{<W;\frac{\psi}{v}>}$$

This new term vanished when the shape function does not depend on time, the static case. For fast transients, if one defines $\rho_g\equiv\rho+\lambda_S\Lambda$ and $\lambda_g\equiv\lambda+\lambda_S$ the point kinetics normal form is restored; but in that case, the kinetic and static measurements of reactivity might differ.

5. The neutron noise procedure

Neutron noise measurements are proposed for reactivity control in large sub-critical reactor. Because reactivity feedback will hardly change the multiplication factor, some procedures useful in zero power reactors could be used. Particularly the Rossi-alpha and the Feynman-alpha techniques seem the most promising procedures for estimating the sub-critical reactivity and the kinetics parameters. For instance, in the case of Feynman-alpha [9]:

$$\frac{\sigma^2(t)}{Z}=1+\varepsilon\frac{D_v}{(\beta-\rho)^2}(1-\frac{1-e^{-\alpha t}}{\alpha t})$$

where Z is the average neutron detector count during the time t, ε is the detector efficiency, σ the standard deviation when the experiment is repeated several times, D_v the Diven factor and $\alpha = (\rho-\beta)/\Lambda$ the parameter to be determined in order to measure the reactivity.

Using such a traditional expression, the reactor should be sub-critical and stationary. The formula is based on the assumption that the neutron source follows the Poisson statistics, i.e. the probability of emitting one neutron in dt is $dt/S^{-1} = Sdt$. Clearly the pulsed spallation source is not a Poisson like source because they are at least two well defined times: the pulse duration and the inverse pulse rate, so some research is still needed determining an equivalent Feynman-alpha expression.

A correction is proposed recently, multiplying the Diven's factor by:$1+SD_S(-\rho)/(\nu D_\nu)$, where D_S is the Diven's factor for the source.

$$D_S = \frac{< S(S-1) >_{aver}}{S^2}$$

but the derivation is based on a continuous spallation source rather than pulsated. Besides, if S were known, the simpler multiplication factor expression is much more convenient determining the k. In spite of being of little practical interest, the expression above alert us about the correction of D_ν, not only because of the spallation source, but also because of ν, which is drifting continuously as a consequence of the fuel burn-up.

As a way out, the Feynman-alpha technique suggest that:

$$\frac{\sigma}{Z} = \sqrt{a + \frac{b}{P}}$$

where a and b are two constants to be fitted, having in mind that the left hand term does not depend on the detector efficiency and the power P can be changed by changing the accelerator pulse rate without affecting to the reactivity.

6. Summary

Some remarks for sub-critical reactors with spallation neutron source are made:

- After fuel reprocessing option, the actinide content will cover the incinerator costs.
- The k_{eff} is going to drift along the reactor operation so new methods for reactivity control are to be found.
- The neutron source is of capital importance for highly sub-critical reactor and cannot be considered as "external".
- The k_{eff} and the others kinetics parameters depend on the source position.
- The shape function of the neutron flux might not be constant along a given fast transient due to the fast pulse rate of the spallation source. If so, static and dynamic measurements of reactivity might differ.
- Neutron noise analysis techniques for controlling reactivity can be used, but the traditional way of doing it should be corrected.

As a conclusion, the ADS option deserves still a lot of research, even for those elemental and well-established concepts.

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